

**Project Report  
ATC-139**

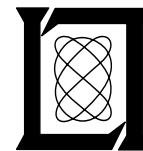
# **A Coordinate Conversion Algorithm for Multisensor Data Processing**

E.M. Shank

5 August 1986

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**Lincoln Laboratory**  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
*LEXINGTON, MASSACHUSETTS*



Prepared for the Federal Aviation Administration,  
Washington, D.C. 20591.

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1. Report No. DOT/FAA/PM-86/26		2. Government Accession No.		3. Recipient's Catalog No. <i>Errata Inserted</i>	
4. Title and Subtitle A Coordinate Conversion Algorithm for Multisensor Data Processing				5. Report Date 5 AUGUST 1986	
7. Author(s) Eric M. Shank				6. Performing Organization Code	
9. Performing Organization Name and Address Lincoln Laboratory, MIT P.O. Box 73 Lexington, MA 02173-0073				8. Performing Organization Report No. ATC-139	
12. Sponsoring Agency Name and Address Department of Transportation Federal Aviation Administration Systems Research and Development Service Washington, DC 20591				10. Work Unit No. (TRAINS)	
				11. Contract or Grant No. DTFA01-84-Z-2-02030	
				13. Type of Report and Period Covered Project Report	
				14. Sponsoring Agency Code	
15. Supplementary Notes The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, under Air Force Contract F19628-85-C-0002.					
18. Abstract  Processing of aircraft surveillance data from several geographically separated radars is most easily accomplished using a common coordinate system to represent data from all sensors. The Multisensor Data Processing system currently being developed for the FAA in support of the Advanced Automation System (AAS) requires a degree of accuracy and consistency that is not available from the current NAS implementation of coordinate conversion. A study has been undertaken to design a coordinate conversion algorithm that meets the needs of Multisensor Data Processing.  The process of projection of the ellipsoidal surface of the earth onto a planar surface is examined in light of the requirements of air traffic control systems. The effects of the non-spherical nature of the earth and of limited computational resources are considered. Several standard cartographic projection techniques are examined, and the stereographic projection is found to be the projection of choice.  A specific implementation of stereographic projection that meets the needs of Multisensor Data Processing is described. This implementation makes use of several approximations to decrease the computational load. The systematic errors introduced by these approximations are removed by the addition of a correction term determined from a precomputed error surface. The performance of this conversion system is demonstrated using realistic test data.					
17. Key Words Coordinate Conversion      NAS Stereographic Projection      AAS Multisensor Aircraft Surveillance			18. Distribution Statement Document is available to the public through the National Technical Information Service, Springfield, VA 22161.		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 50	22. Price

**Unclassified**

ERRATA SHEET

Project Report ATC-139

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In the upper left hand corner this report reads DOT/FAA/PM-86/26. It should now read DOT/FAA/PM-86/37. Please change the cover sheet and title page to reflect this change.

1 October 1986

Publication Office  
M.I.T. Lincoln Laboratory  
P.O. Box 73  
Lexington, MA 02173-0073

**Unclassified**

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## EXECUTIVE SUMMARY

### Introduction

Lincoln Laboratory is currently developing a Multisensor Data Processing system for use by the FAA. This system will make optimum use of all available data from multiple sensors to provide improved tracking of aircraft. The improved data will be of high enough quality to support reliable automated conflict detection and resolution. In order to use data from widely separated radar sensors, a master coordinate system is required, and an algorithm must be designed to convert from measured range and azimuth coordinates to the master coordinates. Because of the high quality required of the data produced by the system, the conversion algorithms currently in use by the National Airspace System (NAS) are not applicable. Therefore, a study was undertaken to design a conversion algorithm that is capable of providing the high accuracy required by the Multisensor Data Processing system within the constraints defined by available real-time computing power.

### Cartographic Projections

Because of the nature of the air traffic control environment, with aircraft spending most of their flying time at a fixed altitude relative to sea level, it is useful to model an air traffic control system in terms of aircraft flying above a flat earth. To match this model, it is appropriate to use planar Cartesian coordinates as the master coordinate system. A variety of standard cartographic projection techniques are available to project sections of the earth's surface onto a plane. A number of these techniques were considered for use.

Any projection that maps a sphere onto a plane will introduce errors, in terms of distortion of linear distances, angles and shapes, or areas. In general, a projection can be designed to avoid distortion in only one of these three, and only over small regions. In the case of air traffic control, particularly for automated conflict detection, aircraft heading errors caused by angular distortion have the most serious negative effect on system performance. Since heading validity is of primary importance, a conformal or angle preserving projection should be used for conversion. Several different conformal projections are available. The choice of the "best" of these projections depends on the geometry of the area to be projected. Of the several standard conformal projections, stereographic projection was found to be the projection of choice on the basis of minimal distance errors over a typical air traffic control coverage area.

### Effects of Non-Spherical Earth Geometry

One of the major requirements for coordinate conversion in the Multisensor Data Processing system is the elimination of multisensor registration errors. Multisensor registration errors occur when the master coordinates of a given aircraft generated by data from one sensor disagree

with those generated for the same aircraft by data from a second sensor. The magnitude of multisensor registration errors due to errors or approximations in the conversion algorithm (exclusive of measurement biases) must be less than about .005 nmi. For a spherical earth, this level of accuracy is not difficult to achieve. However, the earth is not spherical, and use of a spherical model of the earth in coordinate conversion results in errors of the order of .1 nmi. Therefore, it is necessary to use an algorithm that models the earth as an ellipsoid, or to design a method of correcting the errors introduced by the approximations involved in the spherical model. An algorithm that uses an ellipsoidal earth model can be constructed, and is outlined in this memo. However, because it requires iterative solutions of nonlinear equations, the ellipsoidal earth model is not suitable for use in real-time air traffic control systems. Since this model cannot be used, correction is applied to an algorithm based on a spherical earth model.

### Coordinate Conversion for Multisensor Data Processing

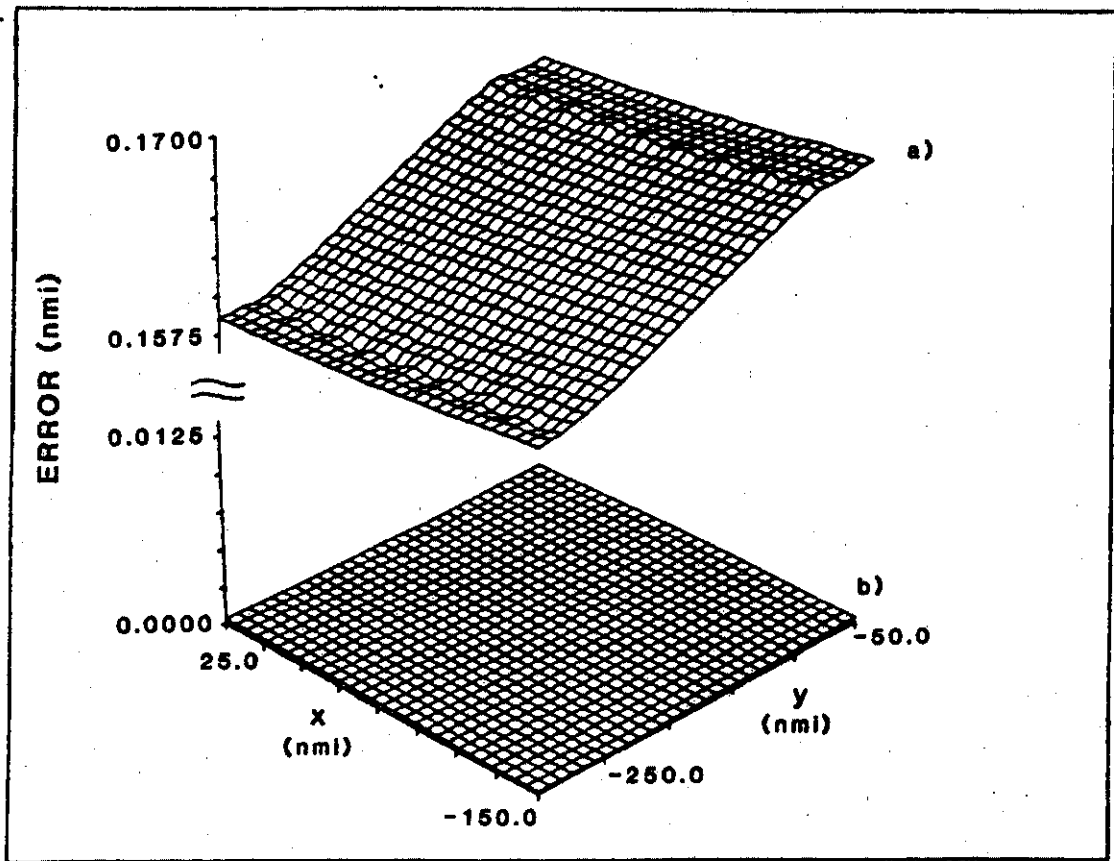
The implementation of stereographic projection designed during this study uses a three step conversion process.

- 1) The measured range, azimuth and altitude are used to determine the target location in a stereographic coordinate system with its origin at the radar site.
- 2) A transformation is made from local radar-centered stereographic coordinates to an initial estimate of the master coordinates.
- 3) A correction is applied to remove errors introduced by use of a spherical earth approximation in calculating the initial master coordinates.

The first two steps are implemented as an improved version of the coordinate conversion algorithm used in the current NAS data processing system. The correction step is carried out by evaluation of a precomputed polynomial approximation to the systematic errors in the system. The errors are determined before system startup by comparing the results of conversion using spherical and ellipsoidal earth models. The differences are computed on a grid around each sensor and a two-dimensional polynomial fit is made to the calculated errors. A correction can then be determined by evaluating the polynomial at the initial estimate of the master coordinates.

This algorithm has been tested using realistic sensor geometries. The results of the tests show that the algorithm meets all anticipated needs of the Multisensor Data Processing system for any realistic air traffic control regional coverage area. Figure ES-1 illustrates the superiority of the newly designed algorithm over the NAS algorithm. Multisensor registration errors are plotted as a function of position for two sensors separated by 100 nmi. The magnitude of the errors in the NAS algorithm are larger by several orders of magnitude.





**Fig. ES-1. Comparison of multisensor registration error a) NAS algorithm b) improved algorithm with correction.**

## 1.0 INTRODUCTION

This report describes a study of coordinate conversion algorithms for use in the Multisensor Data Processing System being developed at Lincoln Laboratory for the FAA. Several possible algorithms are considered and a preferred algorithm is chosen. Implementation of this algorithm is then considered, and a specific implementation that meets the requirements of the Multisensor Data Processing System is described.

### 1.1 Objective

The objective of this work is to develop a method for conversion of aircraft position coordinates from a radar centered system of range, azimuth, and altitude to a master system using x, y, and altitude coordinates. The coordinate conversion method must meet certain performance limits in terms of conversion accuracy, distortion generated by the mapping process, and computational efficiency.

### 1.2 Motivation

In the Multisensor Data Processing System, data from several radars will be used for range-range multilateration and estimation of aircraft altitude and system biases. The purpose of this processing is to provide improved estimates of aircraft position, speed, and heading, and to eliminate offsets due to system biases when a preferred sensor misses an aircraft on one or more scans. This improved information is expected to generate aircraft position and velocity data of a quality that allows use of automated algorithms for conflict detection and resolution.

Such a system requires a valid method of comparing data from the different sensors. In current systems this is done by converting the measured range and azimuth from each radar into a set of master system coordinates that are used for all comparisons. Since most aircraft spend most of their flying time at a constant altitude relative to sea level, it is easiest to visualize a region of air space in terms of aircraft flying at a fixed altitude above a planar (flat earth) coordinate system. As a result, the appropriate master coordinate system for air traffic control is a planar one using x, y, and altitude coordinates.

It is therefore necessary to design an algorithm to convert from the nearly spherical geodetic system of latitude and longitude coordinates to a planar Cartesian coordinate system. The method used to make this conversion has a direct effect on the overall performance of the traffic control system. Because Multisensor Data Processing is intended to provide high quality data for use with automated conflict detection systems, the algorithm used for conversion from local, radar-centered, coordinates to a set of master system coordinates will require accuracy and distortion limits that are more stringent than those for the multiple radar surveillance systems that are operational at present.

There are several specific requirements for such a coordinate conversion algorithm. Ideally, in the absence of sensor noise and/or bias the conversion process should:

1. Map measurements of a single target made by the various sensors to the same system coordinates.
2. Map measurements of two separate targets at the same latitude and longitude, but different altitudes, to the same x and y system coordinates, while maintaining the correct altitudes.
3. Avoid distortions that cause loss of heading validity.
4. Minimize the unavoidable magnification errors caused by mapping the surface of an ellipsoid onto a plane.
5. Be computationally efficient so that the conversion algorithm can be applied to large numbers of aircraft in real time for realistic radar scan rates.

The choice of a conversion method, and the particular implementation of that method, must take into account tradeoffs between several of these requirements. In particular, requirements of conversion accuracy and computational efficiency are often found to be in conflict.

### 1.3 Approach

In light of these requirements a study was undertaken to

1. Determine which of the standard cartographic techniques for projection of an ellipsoid onto a plane would be most appropriate for use in the Multisensor Data Processing System;
2. Develop, if possible, a specific implementation of the chosen projection technique that meets the conversion accuracy and computational efficiency requirements of the Multisensor Data Processing System.

This study lead to the conclusion that the stereographic projection, which is used in the current NAS Multiple Radar Data Processing system, is the best candidate projection technique. Examination of implementation details such as approximations to the exact projection and techniques for correction of projection errors then led to an implementation that meets the accuracy and efficiency requirements.

### 1.4 Outline

The remainder of this report will describe the process leading to these results, as well as the details of the coordinate conversion algorithm that was developed. The next section will provide some background on the process of converting geodetic coordinates to planar coordinates, including brief descriptions of several of the standard cartographic projection techniques. Section 3 discusses the issues involved in the choice of a preferred projection and recommends stereographic projection as the projection technique

of choice. A general discussion of the use of stereographic projection in a multiple radar system is given in section 4. This section also includes a more detailed examination of the implementation of stereographic projection used by the NAS Multiple Radar Data Processing system. Section 5 describes the details of the improved conversion algorithm that was developed during this study. Section 6 provides a summary and conclusions.

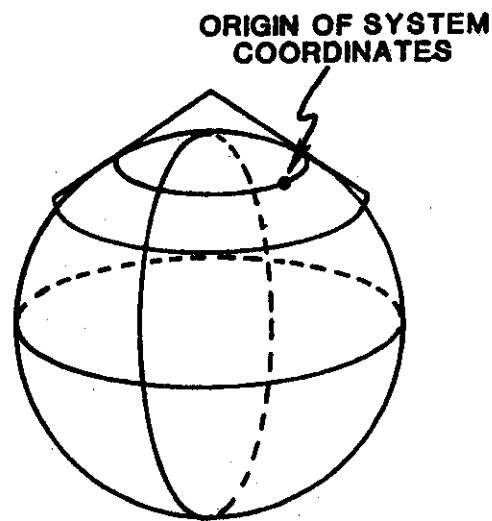
## 2.0 CONVERSION OF GEODETIC TO PLANAR COORDINATES

Conversion of coordinates from geodetic (latitude, longitude, altitude) to Cartesian (x, y, altitude) frames of reference is a difficult process. Among the problems encountered are unavoidable errors such as distortions of shapes or areas and magnification errors, the stretching and compression of space that occurs in mapping a curved surface onto a plane. Additional errors may be caused by approximations introduced to improve the computational efficiency of the algorithms. This section discusses the general nature of the problem of mapping the earth onto a flat surface and describes several of the standard projection methods used by cartographers for that purpose.

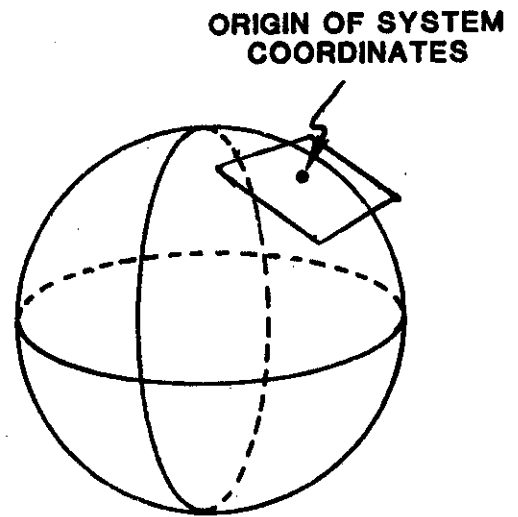
### 2.1 General Nature of the Problem

In general, the problem of coordinate conversion for air traffic control systems is that of mapping a small region of the earth onto a planar surface. The region covered is typically approximately square with each side of the area several hundred miles long. Such a region is large enough that distortions caused by the projection can become significant. There are a variety of methods, developed for use in cartography, that can be used to make such a mapping, and the choice of one method over another depends on the type and magnitude of the distortions introduced by each.

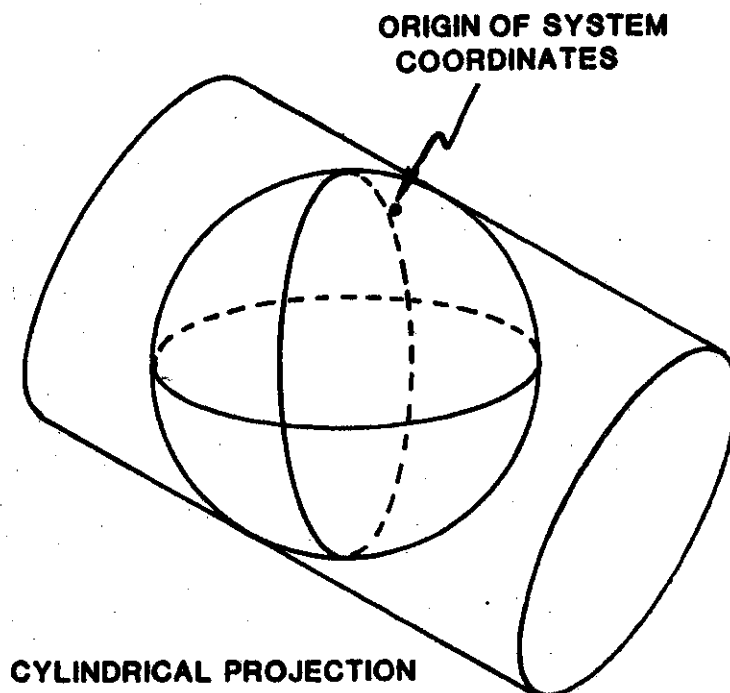
Most mapping or projection techniques can be broadly divided into categories having to do with the geometric basis of the projection and with the specific properties of the original surface that are to be preserved in the projected image. In terms of geometry, most projections can be classified as azimuthal, conical or cylindrical. Azimuthal mappings are projections directly onto a plane surface, while the other mappings are projected onto a cylinder or a cone that is then unwrapped to a flat surface. The placement of these surfaces on the earth is illustrated in Fig. 2.1. Figure 2.1 shows the projection surfaces tangent to the earth at all points of contact. Projections made using tangent surfaces are known as tangent projections. Alternatively, the surfaces can be changed so that they cut through the earth's surface. This type of projection is called a secant projection, and the effect on the projected planar coordinates is the same as the application of an overall multiplicative scale factor to a tangent projection of the same type. Projections using the different types of projection surface are best suited to different types of regions to be mapped. In general, a cylindrical projection is most appropriate when the area to be mapped is rather long and narrow, and runs along a great circle of the earth. An azimuthal projection is most useful when applied to a fairly small, roughly square or circular region of the earth's surface, and conical projections are most useful for narrow regions running parallel to a small circle of the earth (a parallel of latitude for example). Since in air traffic control, the region to be mapped is generally approximately square and of relatively small size, some type of azimuthal projection may be expected to be most appropriate.



**CONICAL PROJECTION**



**AZIMUTHAL PROJECTION**



**CYLINDRICAL PROJECTION**

**Fig. 2-1. Projection surfaces.**

Projections can also be classified as equidistant, conformal or equivalent, depending on what properties of the original surface are preserved on the mapped surface. An equidistant projection preserves the length (scale) of line segments along certain preferred directions, while conformal mappings preserve shape and direction and equivalent projections preserve area.

In air traffic control systems, the important quantities are distance and direction. In general, it is more important to preserve direction and thereby the validity of aircraft heading than to have aircraft separations exactly correct, particularly since magnification errors are typically a maximum of one or two percent, and approach zero as the actual separation approaches zero. In light of this, and keeping in mind that distances are preserved only along certain special directions even in equidistant projections, it is desirable to use a conformal or shape-preserving projection for coordinate conversion in air traffic control systems.

Several conformal projection algorithms will be discussed and evaluated in some detail later. These projection methods are based on a model of the earth that describes the earth as a sphere. In fact the earth is not spherical, but ellipsoidal in shape. The deviation of the shape of the earth from a sphere is small, about 12 nautical miles difference between the polar and equatorial radii, but can introduce significant errors in a projection algorithm based on a spherical earth model. The effect of the non-spherical earth on projection methods for use in air traffic control, and methods for correcting the errors caused by using a spherical earth model will also be examined in later sections.

The process of mapping aircraft positions onto a set of Cartesian system coordinates is complicated by the fact that standard projection methods use latitude and longitude as input. Aircraft positions, on the other hand, are measured in terms of range and azimuth angle from a specified location. Altitude from an on-board altimeter is sometimes, but not always, known as well. This creates an incompatibility between the measured quantities and the required inputs for projection to x, y position using the standard projection techniques. This incompatibility can be handled in two ways. The correct geodetic coordinates can be determined by calculating, using range, azimuth and altitude, the latitude and longitude of the point of intersection between the surface of the earth and a line perpendicular to the earth's surface drawn through the aircraft. The cartographic projection techniques can then be applied directly to determine the x, y position in the master coordinate system. Alternatively, the range, azimuth and altitude can be used to determine the position of the aircraft in a local Cartesian coordinate system. This process is easier and less costly computationally than calculation of the geodetic position of the aircraft. Once the position in the local plane is known, the cartographic projection techniques can be used to determine a set of equations that transform the local x, y coordinates into master plane coordinates, given knowledge of master plane parameters and the radar site location on the master plane. The transformation equations used in this process are usually complicated and computationally inefficient. For this reason, series approximations to the equations are used. This leads to very efficient computer algorithms, but usually introduces additional errors from approximations such as truncation of infinite series. The tradeoffs between these procedures will be examined when specific algorithm implementation is discussed.

## 2.2 Cartographic Projection Techniques

In this section, three candidate projection techniques are described. For reasons discussed previously, all three are conformal mappings designed to preserve direction at the expense of somewhat increased magnification errors. The brief descriptions given include the mathematical formulas that are used to apply the projection as well as geometrical descriptions of the projection when such descriptions are appropriate. For a more complete treatment of these projections, see Thomas<sup>1</sup> or Richardus and Adler<sup>2</sup>. The formulas used for projection into local x, y coordinates and transformation to master plane coordinates are also included. These formulas are given by Goldenberg and Wolf<sup>3</sup>. In the formulas, the following definitions are used.

E	=	radius of the spherical earth
L	=	latitude of the projected point
$\lambda$	=	longitude of the projected point
$L_0$	=	latitude of the origin of coordinates in the projection
$\lambda_0$	=	longitude of the origin of coordinates in the projection
$\Delta\lambda$	=	$(\lambda_0 - \lambda)$ when longitude is west of Greenwich
H	=	aircraft altitude (above sea level)
h	=	radar site altitude (above sea level)
$\theta$	=	aircraft azimuth measured at radar site
$\rho$	=	aircraft range measured from radar site
x, y	=	coordinates of a point in the master Cartesian system
u, v	=	coordinates of a point in the radar-centered Cartesian system

The projection techniques described here are the oblique Mercator projection, the stereographic projection and the Lambert conformal conic projection.

### 2.2.1 The Oblique Mercator Projection

The oblique Mercator projection is a conformal, cylindrical projection. The projection from geodetic coordinates to the unfolded cylindrical surface is made using

$$\begin{aligned}
 x &= E \arctan \frac{\sin\Delta\lambda \cos L}{\cos L \sin L_0 \cos\Delta\lambda - \cos L_0 \sin L} \\
 y &= -\frac{E}{2} \ln \frac{1 + \sin L \sin L_0 + \cos L \cos L_0 \cos\Delta\lambda}{1 - \sin L \sin L_0 - \cos L \cos L_0 \cos\Delta\lambda} \quad (2.1) \\
 &= E \operatorname{arctanh} (\sin L \sin L_0 + \cos L \cos L_0 \cos\Delta\lambda)
 \end{aligned}$$

When the mapping is made to a projection with the origin of coordinates at the radar position and then transformed to a master projection having a different origin, the projection to the local Cartesian coordinate system is made by

$$\begin{aligned}
 u &= E \arctan (\sin \theta \tan \phi) \\
 v &= E \operatorname{arctanh} (\cos \theta \sin \phi) \quad (2.2) \\
 \phi &= \arccos \frac{(E+h)^2 + (E+H)^2 - \rho^2}{2(E+h)(E+H)}
 \end{aligned}$$



and the expression for transforming to the master projection is

$$\tan \frac{W}{2E} = \frac{e^{i\beta} \tan \frac{Z}{2E} + \tan \frac{W_0}{2E}}{1 - e^{-i\beta} \tan \frac{Z}{2E} \tan \frac{\bar{W}_0}{2E}} \quad (2.3)$$

where

- $W$  = target location in complex master coordinates =  $x + iy$
- $Z$  = target location in complex local coordinates =  $u + iv$
- $\beta$  = angle between true north and master y-axis at radar site
- $W_0$  = radar location in complex master coordinates =  $x_0 + iy_0$
- $\bar{W}_0$  = complex conjugate of  $W_0$  =  $x_0 - iy_0$

The value of  $\beta$  is calculated as

$$\beta = \frac{(\sin L_0 + \sin L) \sin \Delta \lambda}{\cos \Delta \lambda + \sin L_0 \sin L \cos \Delta \lambda + \cos L_0 \cos L}$$

As was previously mentioned, this expression is not used directly in time critical operations. A Taylor expansion around  $Z = 0$  can be used to generate a rapidly converging series approximation to the exact expression.

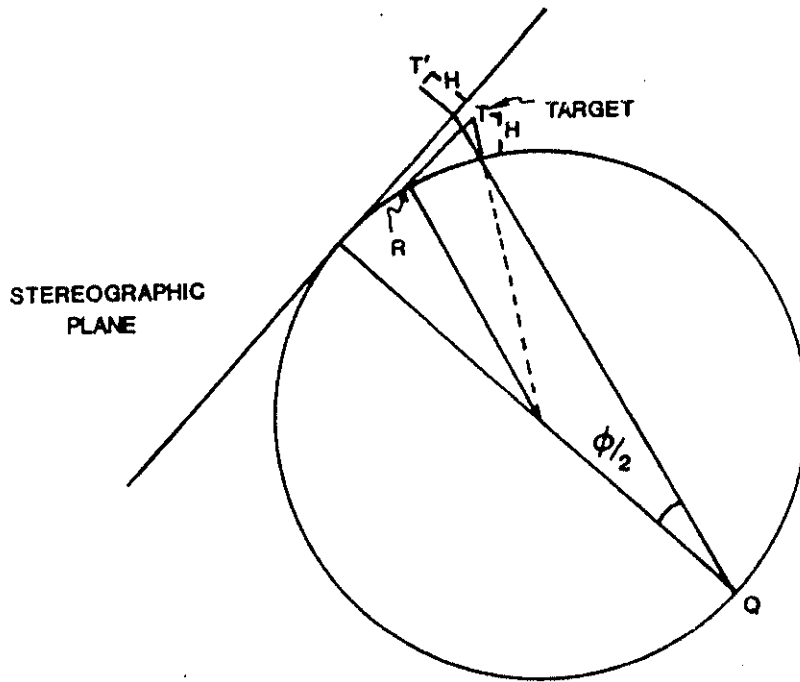
A geometrical representation of the Mercator projection is not possible. As a true conformal mapping to a cylinder, the Mercator projection requires a spacing of the coordinate axes perpendicular to the cylinder axis that cannot be produced by a direct geometric projection.

### 2.2.2 The Stereographic Projection

The stereographic projection is a conformal, azimuthal projection. The geometrical representation for the projection is shown in Figure 2.2. As seen in the figure, the origin of stereographic coordinates is at the point of tangency of the projection plane, and the perspective point is diametrically opposite to the point of tangency. The projection is made by passing a straight line through the perspective point and the point on the surface of the earth that is to be projected onto the plane. The stereographic position is determined by the point where this line segment intersects the projection plane.

The direct projection is defined mathematically by

$$x = 2E \frac{\sin L \cos L_0 - \cos L \sin L_0 \cos \Delta \lambda}{1 + \sin L \sin L_0 + \cos L \cos L_0 \cos \Delta \lambda} \quad (2.4)$$



- T' - TARGET POSITION PROJECTED ONTO THE STEREOGRAPHIC PLANE
- R - RADAR LOCATION
- Q - PERSPECTIVE POINT FOR PROJECTION

Fig. 2-2. Geometry of the stereographic projection

$$y = 2E \frac{\cos L \sin \Delta \lambda}{1 + \sin L \sin L_0 + \cos L \cos L_0 \cos \Delta \lambda}$$

Projection into a local stereographic plane with the point of tangency at the sensor location is made using

$$u = D \sin \theta$$

$$v = D \cos \theta$$

$$\phi = \arccos \left[ \frac{(E+h)^2 + (E+H)^2 - \rho^2}{2(E+h)(E+H)} \right] \quad (2.5)$$

$$D = 2E \tan \left( \frac{\phi}{2} \right)$$

Note that this projection process also assumes that the earth can be approximated locally as a sphere. The expression

$$W = \frac{Ze^{-\beta} + W_0}{1 - Ze^{-\beta} \overline{W}_0} \quad (2.6)$$

$$(2E)^2$$

is then used to transform from the local stereographic plane to the master plane. Here  $W$ ,  $W_0$ ,  $\overline{W}_0$ ,  $Z$ , and  $\beta$  are as defined in section 2.2.1. Once again, this last expression is not very useful in time critical operations, and an expansion can be made to provide a rapidly converging series for use in actual calculations.

### 2.2.3 The Lambert Conformal Conic Projection

As is evident from the name of this projection, it is a conformal projection onto a cone. The direct projection from the earth to the cone is made using

$$y = E(\cot L_0) r_0^{\ell} \sin(\ell \Delta \lambda) \quad (2.7)$$

$$x = E(\cot L_0) (1 - r_0^{\ell} \cos(\ell \Delta \lambda))$$

where

$$r = \frac{\cos L_0 (1 - \sin L)}{\cos L (1 - \sin L_0)}$$

$$\ell = \sin L_0$$

Figure 2.3 demonstrates the geometrical description of this projection. The origin of coordinates lies on the circle of contact with the earth, and the perspective point is the center of the earth. As with stereographic projection, the projected image position is determined by the intersection of the projection surface and a line drawn between the perspective point and the point on the surface of the earth that is being projected.

Projection onto a surface with the origin of coordinates at the radar surface is described mathematically by

$$\begin{aligned}
 u &= D \sin \theta \\
 v &= D \cos \theta \\
 D &= E \tan \phi
 \end{aligned}
 \tag{2.8}$$

with  $\phi$  as defined in Eq. (2.2).

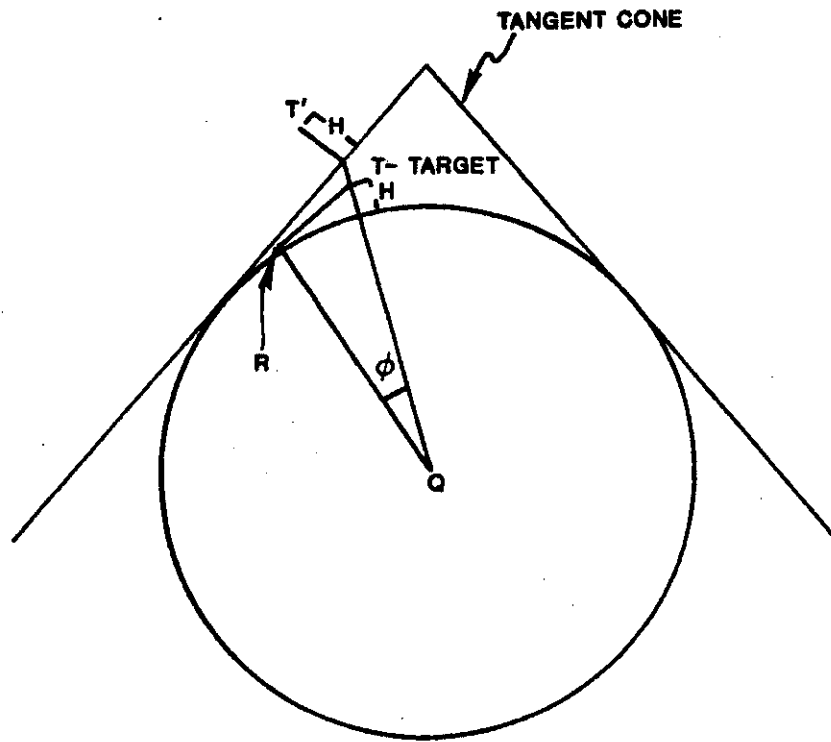
The transformation equation for this projection is

$$W = iE(\cot L_0)(1 - \exp[\ell(\tau_0 - \tau) + i\ell(\lambda_0 - \lambda)])
 \tag{2.9}$$

where

$$\begin{aligned}
 \tau_0 &= \ln \left( \frac{\cos L_0}{1 - \sin L_0} \right) \\
 \tau &= \ln \left( \frac{\cos L}{1 - \sin L} \right)
 \end{aligned}$$

Once again a series expansion can be made to allow more efficient computation.



**T' - TARGET POSITION PROJECTED ONTO THE TANGENT CONE**

**R - RADAR LOCATION**

**Q - PERSPECTIVE POINT FOR PROJECTION**

**Fig. 2-3. Geometry of the Lambert conic projection.**

### 3.0 CHOICE OF A CONVERSION TECHNIQUE

The choice of a specific conversion technique for use in air traffic control applications must be based on a set of requirements that are designed to minimize the impact of any distortions on the quality of the data supplied to the controller's display. From earlier discussion it is clear that the performance of any given technique will be affected by the geometry of the area to be covered by a control region. Since control regions are typically approximately square, evaluations of conversion performance will be made using a square coverage area. In addition, the amount of computation involved in the application of a given technique must be examined. The final choice of a technique must be made considering the magnitude of distortions introduced, their impact on the quality of data generated, and the computational cost of the chosen system. In this section, a set of requirements for general air traffic control applications is defined, and the projection techniques described in Section 2 are evaluated in light of these requirements. On the basis of this evaluation, stereographic projection is recommended as the projection of choice.

#### 3.1 Requirements for Coordinate Conversion

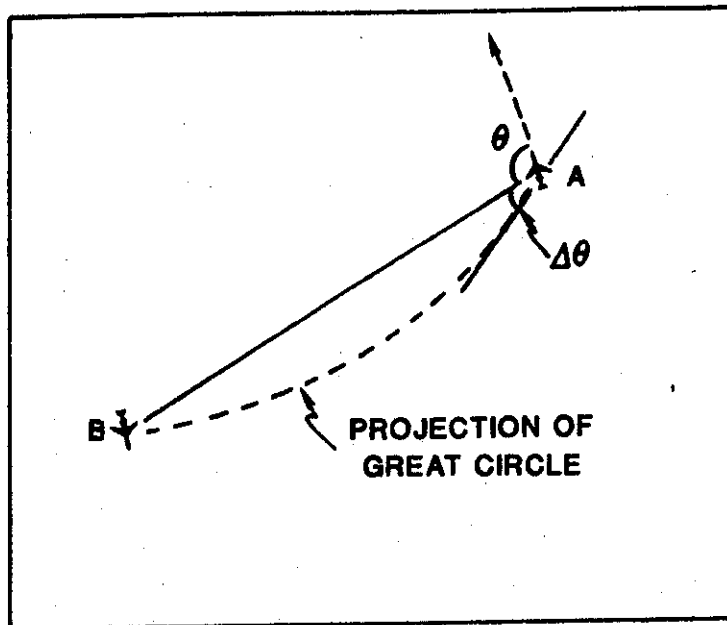
In order to provide useable data to air traffic controllers in terms of a set of Cartesian system plane coordinates, the method used to convert from geodetic coordinates must meet certain requirements. The conversion algorithm must:

- 1) Maintain the validity of measured aircraft heading.
- 2) Be altitude independent.
- 3) Minimize computation time.
- 4) Minimize magnification errors.

The projection method of choice is the one that best satisfies these requirements.

##### 3.1.1 Heading Validity

The requirement that heading validity must be maintained restricts the choice of projection method to the class of conformal projections. As discussed earlier, a conformal projection preserves angles and shapes during the conversion from the earth's surface to the system plane representation. In particular, if two lines drawn on the surface of the earth intersect at an angle of 30 degrees, the conformal projection of those two lines onto a planar surface will also intersect at a 30 degree angle. This in itself does not guarantee that the bearing angle of one aircraft relative to another, as calculated in the system plane, will be the same as the actual angle in an earth-based coordinate system. This is because, as illustrated in Fig. 3.1, a "straight" line on the earth's surface (i.e., a great circle line) may not map to a straight line in the projection. However, over a small area, the curvature of great circle lines on a conformal projection is negligible. In



$\theta$  = BEARING CALCULATED IN STEREOGRAPHIC COORDINATES  
 $\theta + \Delta\theta$  = ACTUAL BEARING AS MEASURED ON THE EARTH

Fig. 3-1. Error in calculated bearing angle.

an area the size of a typical ATC regional coverage area, headings calculated in the system plane will be correct, and bearing angles will be accurate to within about .5 degrees. The errors decrease as separation decreases, and for aircraft separations of 30 nmi or less, the errors in bearing angles are less than .05 degrees. Since this accuracy meets the most stringent anticipated requirement, that of 1 degree heading accuracy for support of automated conflict detection, errors from this source are acceptable.

The alternative is use of a gnomonic projection in which all great circles are mapped to straight lines. This projection is not conformal, and does not preserve angles. Therefore headings and bearing angles calculated in system coordinates are often incorrect, typically with errors larger than those generated by curved mapping of great circle lines in conformal mappings. In addition, these errors do not generally decrease significantly as separation distances approach zero.

### 3.1.2 Altitude Independence

Altitude independence, as specified by requirement 2), means that the system coordinates generated for an aircraft depend only on the latitude and longitude location of the aircraft. If the projection is altitude independent, two aircraft at the same latitude and longitude but different altitudes will map to the same system coordinates and will retain the correct altitudes. Since all of the projections being considered map a point on the sea-level surface of the earth onto a plane, and that point is defined in terms of latitude and longitude only, requirement 2) will be satisfied by any one of the projection methods. It should be noted that some approximations to these projections may not be altitude independent. This should be kept in mind when approximate implementations are considered.

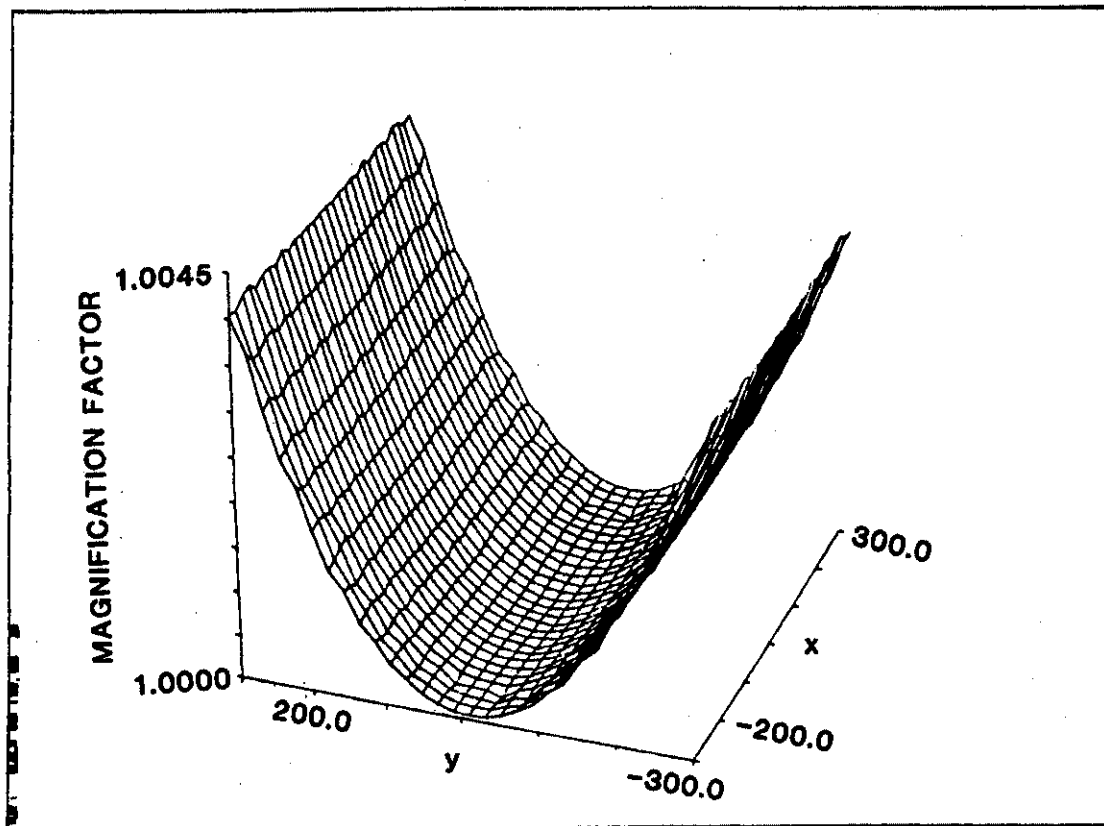
### 3.1.3 Computational Complexity

An examination of the projection equations supplied in Section 2.2 shows that there is little basis for decision in terms of computational complexity between the three algorithms being considered. The expressions for the Stereographic and Mercator projections are quite similar, and though the expressions for the Lambert conic appear to be somewhat simpler, in terms of complicated math functions such as trigonometric and power functions, the level of complexity is little different. Likewise the approximate solutions using projection to a local plane followed by transformation involve comparable computational complexity.

### 3.1.4 Magnification Errors

On the basis of the preceding discussion, the only requirement for which a significant difference is expected to be found is in the magnitude of the magnification errors. Therefore, the performance of the three projection techniques in terms of magnification errors is examined closely to give guidance in the choice of a recommended algorithm. Figures 3.2 through 3.4 show numerical evaluations of magnification errors for the three different projections. The data in the figures was generated using a system plane origin at 40 degrees north latitude. Results using system plane origins at





**Fig. 3-2. Magnification of the Mercator projection.**

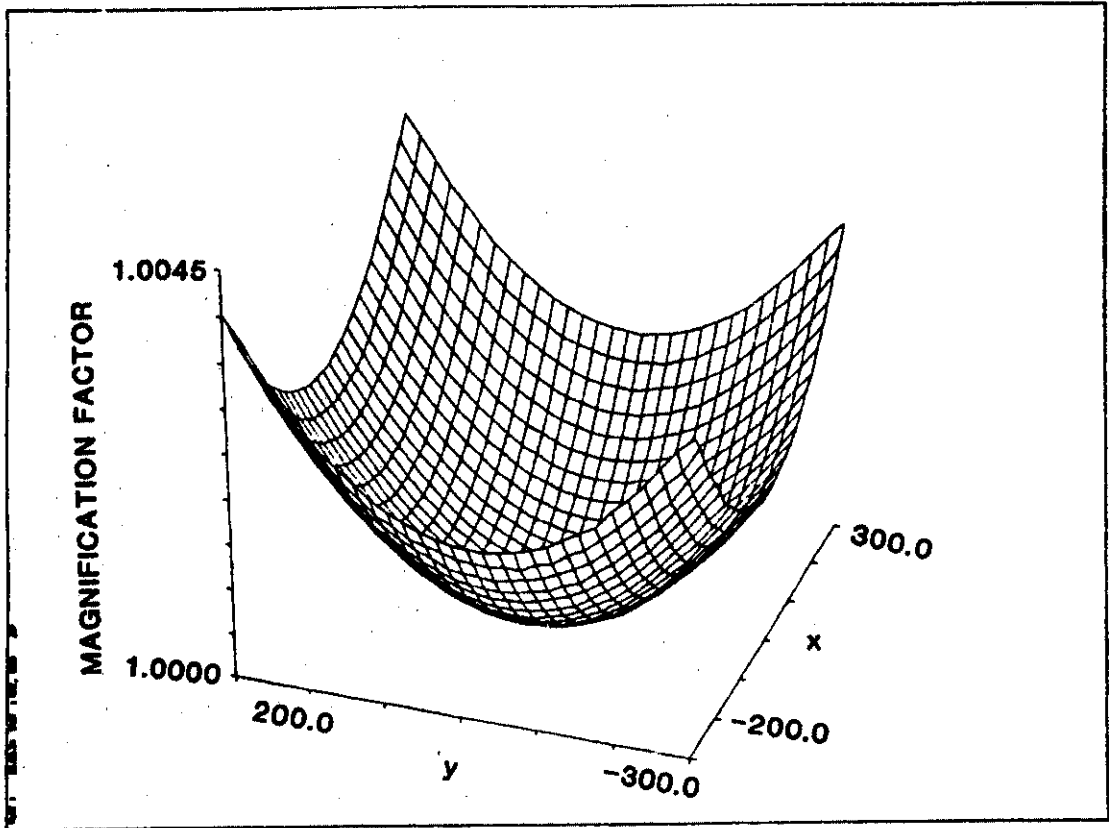


Fig. 3-3. Magnification of the stereographic projection.

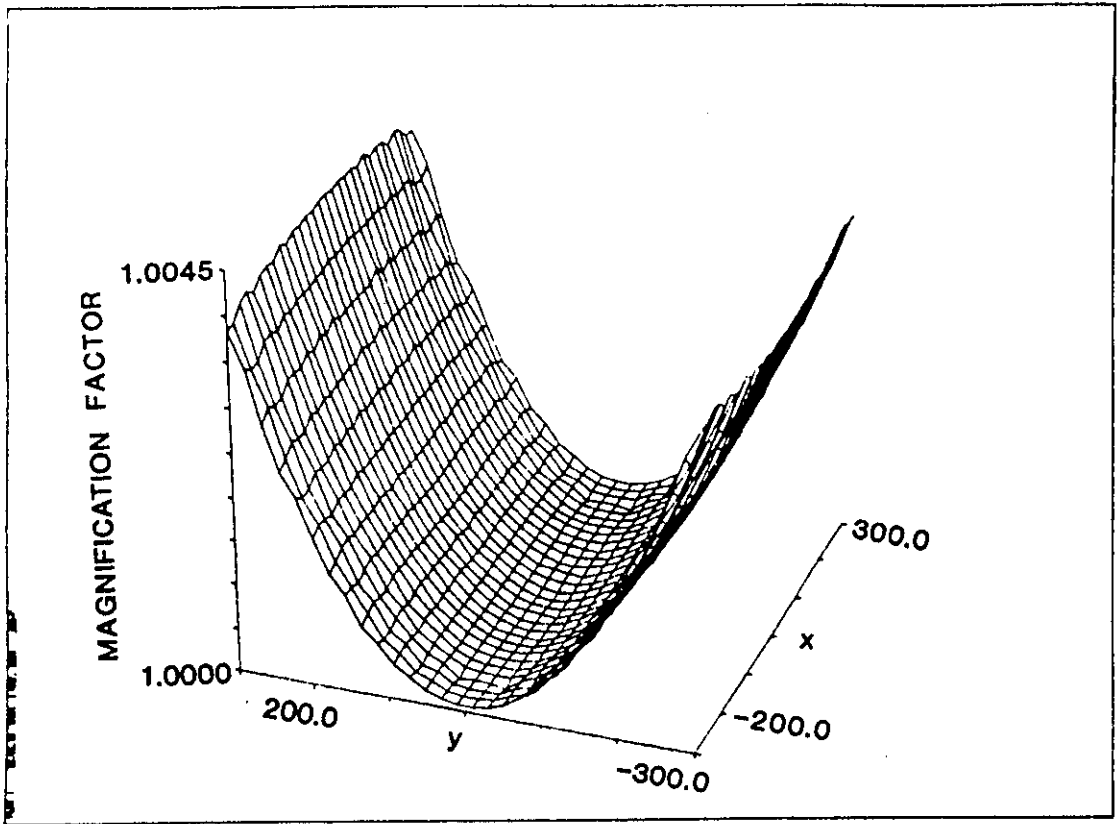


Fig. 3-4. Magnification of the Lambert conic projection.

30 degrees and 50 degrees north latitude do not differ significantly from the plotted data.

The values plotted are the magnification factor as calculated for a grid of points. The magnification factor is the ratio of the length of the projection of a line segment in the system plane to the length of the actual line segment on the earth. In this case, magnification is calculated using the projection of a line segment approximately one nautical mile in length centered on the grid point and oriented in the north-south direction. One property of conformal projections is that the magnification factor at any given point is independent of direction so that the orientation is unimportant. The "true" distances are determined using a calculation that determines the geodesic arc length (for an ellipsoidal earth) to an accuracy of 30 feet (see Kayton and Fried<sup>4</sup>).

Figure 3.2 is a plot of magnification as calculated from the Mercator projection. Figure 3.3 is a similar plot for stereographic projection, while the magnification for the Lambert conformal conic projection is shown in Fig. 3.4. From the figures, it can be seen that there is no single "best" projection for all possible system plane geometries. Either the Mercator or Lambert projection is better if the system coverage area is significantly longer in the east-west direction than in the north-south direction, for example. When the linear extent of the coverage area in the north-south direction is larger than the east-west extent, the stereographic projection is preferred. This is due to the way the projections are constructed. The Mercator and Lambert projections each have a line of contact with the earth's surface. Along this line, the magnification factor is held at unity, but away from the line of contact the magnification error grows. The stereographic projection has only a single point of contact at which the magnification factor is held at unity, while it is larger everywhere else. On the other hand, the magnification factor for the stereographic projection grows only about half as fast as for the other projections as the distance from the points of contact with the earth grows.

### 3.2 Projection Technique of Choice for Air Traffic Control

If a square region of coverage is considered, the significance of choosing an algorithm on the basis of magnification factor becomes clear. If the magnification factor is averaged over the region, the result is very nearly the same for all of the algorithms. Likewise, the maximum and minimum magnification factors are about the same. However, at a given radial distance from the system plane origin of coordinates, the stereographic projection produces the smallest maximum magnification factor, and this magnification is the same regardless of direction from the origin. This is a small, but significant advantage of the stereographic projection over the other projection methods. Since it is desirable to use a single algorithm for all ATC applications, the stereographic projection is chosen as the projection method most appropriate for use in air traffic control.

#### 4.0 STEREOGRAPHIC PROJECTION FOR MULTIPLE RADAR TRACKING SYSTEMS

As mentioned earlier, the various projection techniques can be implemented in a number of different ways, depending on the needs of the particular system in question in terms of conversion accuracy, conversion speed, and system plane size. Stereographic projection is no exception. This section describes the implementation of stereographic projection currently used in the NAS system. The coordinate conversion requirements of the Multisensor Data Processing system currently being developed are discussed and it is seen that the current NAS implementation falls short of these requirements. The performance of an implementation of the exact stereographic projection applied to a realistic system of radars using a locally spherical earth approximation is then examined. It is shown that the use of this approximation introduces unacceptable errors in aircraft locations. Finally, a method of correction for these errors is described.

##### 4.1 Stereographic Projection in the National Air Space System

The coordinate conversion method currently used in the NAS system is an approximation to stereographic projection. The implementation is designed to maintain the errors introduced by the approximations at a level less than the 1/8 nmi range resolution of the NAS enroute sensors. The process uses conversion to a local x,y coordinate system followed by transformation, using Eq. (2.6), into the master Cartesian coordinate system. Both stages of this process use approximations, and these will be described briefly here. For more detailed information see Goldenberg and Wolf<sup>3</sup>, Mulholland and Stout<sup>5,6,7</sup>, Gingerich<sup>8</sup> and Appendix A of NAS publication MD-320<sup>9</sup>.

##### 4.1.1 Conversion in the NAS System

The general process used in conversion of geodetic coordinates to a radar centered x,y coordinate system is illustrated in Fig. 4.1. The exact conversion, assuming a spherical earth, can be expressed (see Goldenberg and Wolf) as

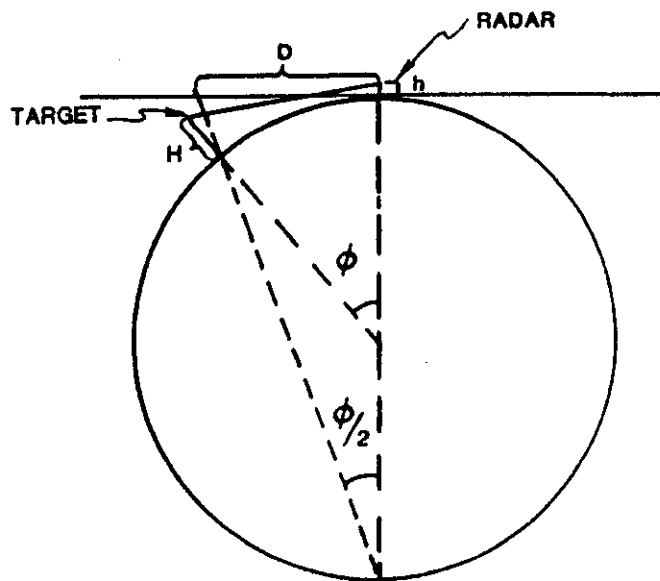
$$u = D \sin \theta \tag{4.1}$$

$$v = D \cos \theta$$

with

$$D = 2E \tan (\phi/2)$$

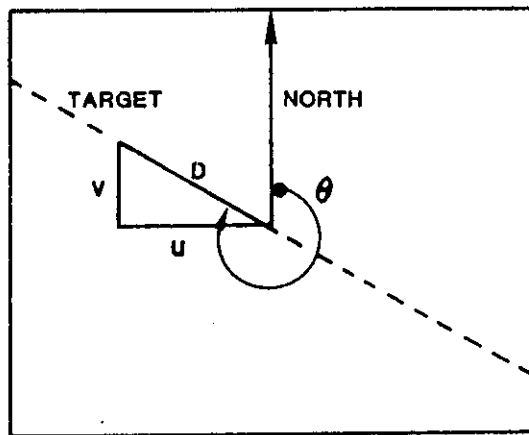
$$= R \left[ 1 + \frac{EH + Eh + Hh}{E^2} - \frac{R^2}{4E^2} \right]^{-1/2} \tag{4.2}$$



$$D = 2E \tan\left(\frac{\phi}{2}\right)$$

$$\cos \phi = \frac{(E+h)^2 + (E+H)^2 - \rho^2}{2(E+h)(E+H)}$$

(A) CALCULATION OF GROUND RANGE, D



$$u = D \sin \theta$$

$$v = D \cos \theta$$

(B) CALCULATION OF LOCAL  $u$  AND  $v$  FROM  $D$  AND AZIMUTH

Fig. 4-1. Conversion to local stereographic coordinates.

where

$$R^2 = \rho^2 - (H-h)^2 \quad (4.3)$$

and  $\theta$ ,  $\rho$ ,  $\phi$ ,  $H$ , and  $h$  are as defined in section 2.2.

In the NAS system, equation (4.2) is not used. Instead, to increase the efficiency of the conversion process,  $D$  is approximated as

$$D = R = \sqrt{\rho^2 - (H-h)^2} \quad (4.4)$$

For an aircraft at a range of 100 nmi, an azimuth of 90 degrees, and an altitude of 35,000 ft, the value of  $D$  from Eq. (4.2) is 99.76 nmi while from the approximation in Eq. (4.4) it is 99.83 nmi. In this case, the error introduced by the approximation is .07 nmi.

#### 4.1.2 Transformation in the NAS System

The formula for transformation from one stereographic plane to another is given in Eq. (2.6). As mentioned earlier, this expression is expensive in terms of computation time, when used in this form. However, a series expansion can be performed to reduce this expression to a rapidly converging infinite series. It can be seen that Eq. (2.6) has the form.

$$W = \frac{A}{1 - B} \quad (4.5)$$

where

$$A = (Ze^{-i\beta} + W_0)$$

$$B = \left( \frac{Ze^{-i\beta} \bar{W}_0}{4E^2} \right)$$

For all realistic system geometries,  $B \ll 1$ , and this expression can be expanded as

$$W = A(1+B+B^2+\dots) \quad (4.6)$$

When terms greater than second order in  $Z$  are dropped, this becomes

$$W = W_0 + T Ze^{-i\beta} + TKZ^2 e^{-2i\beta} e^{-i\gamma} \quad (4.7)$$

where

$$T = 1 + \frac{W_0 \bar{W}_0}{4E^2}$$

$$K = \frac{(x_o^2 + y_o^2)^{1/2}}{4E^2} \quad (4.8)$$

$$W_o \bar{W}_o = (x_o^2 + y_o^2)$$

$$\gamma = \arctan (x_o/y_o)$$

A further simplification is possible since

$$Z = De^{i\theta} \quad (4.9)$$

where

$$\theta = \arctan (u/v)$$

$$D = (u^2 + v^2)^{1/2}$$

From these expressions it can be seen that D and  $\theta$  correspond to the ground range from Eq. (4.2) above and the measured radar azimuth respectively. Substituting for z in Eq. (4.7) and changing to real form gives the result

$$\begin{aligned} x &= x_o + TD \sin (\theta - \beta) + KTD^2 \sin (2(\theta - \beta) - \gamma) \\ y &= y_o + TD \cos (\theta - \beta) + KTD^2 \cos (2(\theta - \beta) - \gamma) \end{aligned} \quad (4.10)$$

In the above expressions W,  $W_o$ ,  $\bar{W}_o$ , and Z are as defined in Section 2.2. This formula is used by the NAS system for transformation. In the current NAS implementation, the second-order terms in this expression are used only when their magnitude may be greater than the 1/8 nmi range resolution of the NAS radars. This requirement defines a range limit, R, for each radar.

$$R = \sqrt{(\sqrt{2}/8)/KT}$$

When the measured range is less than R, only the first order terms in the expansion are used, while the second-order terms are also calculated when the range is greater than R. This guarantees that for any realistic coverage area, the error introduced by the transformation equations will be less than 1/8 nmi.

#### 4.2 Additional Requirements for Multisensor Data Processing

There are two additional requirements for the Multisensor Data Processing system now being developed. The system must

- 1) Use, without significant loss of resolution, the data provided by Mode-S sensors when they come on line.
- 2) Generate identical system coordinates for a given aircraft from measured data obtained from each of the several sensors in a given system, assuming sensor bias errors have been separately handled.



The current NAS implementation of stereographic projection does not meet either of these additional requirements.

The Multisensor Data Processing system now under development is intended to operate during and after the Mode S sensor phase-in period. It is also expected to provide a base on which the FAA Advanced Automation System (AAS) will be built. In light of this it is important for the coordinate conversion algorithms designed for Multisensor Data Processing to be able to use the added accuracy of the data generated by Mode S sensors. Therefore, the projection process should not introduce errors of more than 1/196 nmi. Clearly, the current NAS coordinate conversion algorithm is not capable of this level of performance.

The use made of the radar data in the new system is also fundamentally different from that in the current NAS system. The current system uses data from the preferred sensor as long as that data is available. It is only when data from the preferred sensor is not available that data from an additional radar is used by the system for replacement. In the new MDP system, data from two or more radars will routinely be used for each track update. Therefore, it is very important for the new system that the system plane coordinates generated by the various sensors for a given target are as nearly as possible exactly the same. This requirement is not fulfilled by the current NAS implementation either. In fact, it will be seen in the next section that any implementation of stereographic projection based strictly on a locally spherical earth model will not meet this requirement.

#### 4.3 Performance of Stereographic Projection

Stereographic projection works by projecting locations on the surface of the earth onto a plane tangent to the earth at some point. In particular, the exact expressions from Eq. (2.4) for stereographic projection use latitude and longitude on a spherical earth as input. The form of the earth is not spherical, but rather ellipsoidal. The conformal nature of the stereographic projection is preserved by first making a conformal mapping of the ellipsoid onto a sphere and then applying the usual stereographic projection process.

The formula that describes the conformal mapping of a spheroid onto a sphere is

$$L_c = 2 \arctan \left[ \left( \frac{1 - \epsilon \sin L}{1 + \epsilon \sin L} \right)^{\epsilon/2} \tan \left( \frac{\pi}{4} + \frac{L}{2} \right) \right] - \frac{\pi}{4} \quad (4.11)$$

Using this mapping, and knowing the correct latitude and longitude of the point to be projected gives the exact conformal mapping of the desired point onto the system plane.

Determining the correct latitude and longitude for an aircraft above an ellipsoidal earth, given only the range, azimuth and altitude from a specified site, is a non-trivial problem. The problem can be solved to arbitrary accuracy by solution of the set of simultaneous, nonlinear equations given below.

$$\begin{aligned}
\sigma\beta_1 + \beta_1 \sqrt{\rho^2 - D^2} - D\gamma_1 &= \zeta \cos L \sin \lambda \\
\sigma\beta_2 + \beta_2 \sqrt{\rho^2 - D^2} - D\gamma_2 &= \zeta \cos L \cos \lambda \\
(\sigma - \epsilon^2)\beta_3 + \beta_3 \sqrt{\rho^2 - D^2} + D\gamma_3 &= (\zeta - \epsilon^2) \sin L
\end{aligned} \tag{4.12}$$

where

$$\begin{aligned}
\sigma &= (A_0 + h) & A_0 &= a / \sqrt{1 - \epsilon^2 \sin^2 L_0} \\
\zeta &= (A + H) & A &= a / \sqrt{1 - \epsilon^2 \sin^2 L} \\
\beta_1 &= \cos L_0 \sin \lambda_0 & \gamma_1 &= \cos \theta \sin L_0 \sin \lambda_0 + \sin \theta \cos \lambda_0 \\
\beta_2 &= \cos L_0 \cos \lambda_0 & \gamma_2 &= \cos \theta \sin L_0 \cos \lambda_0 - \sin \theta \sin \lambda_0 \\
\beta_3 &= \sin L_0 & \gamma_3 &= \cos \theta \cos L_0
\end{aligned}$$

and

$a$  = equatorial radius of the earth

$b$  = polar radius of the earth

$\epsilon$  = ellipticity of the earth,  $\epsilon^2 = (1 - \frac{b^2}{a^2})$

Because this solution is an iterative one, and because of the amount of calculation necessary to find the solution, this method of determining the appropriate latitude and longitude for use in the projection process is not useful in a real-time system. Therefore, it is necessary to make some kind of approximation to simplify the process of determining latitude and longitude. The most obvious approximation to apply is to assume that the earth is really spherical, and that it has a radius equal to the average radius of curvature of the actual surface of the earth at the radar site that generated the data to be converted. When this approximation is made, latitude and longitude can be calculated in closed form using standard trigonometric functions and the principles of spherical geometry. These values can be calculated from the expressions

$$\begin{aligned}
L &= \frac{\pi}{2} - \arccos (\sin L_0 \cos \phi + \cos L_0 \sin \phi \cos \theta) \\
\lambda &= \lambda_0 - \arccos \left( \frac{\cos \phi - \sin L \sin L_0}{\cos L \cos L_0} \right)
\end{aligned}$$

where  $S$ ,  $\theta$ ,  $L$ ,  $\lambda$ ,  $H$ ,  $L_0$ ,  $\lambda_0$ ,  $\phi$ , and  $h$  are as previously defined, and  $E$  is the average radius of curvature of the earth at the radar site.

Use of this approximation introduces errors in the form of differences between the projected system plane position and the system plane position that would be generated if the exact latitude and longitude coordinates were known and used for projection. For a given system plane point of tangency and a given radar location, these errors can be evaluated numerically, resulting in an error surface for the region near the radar. Figure 4.2 is a representation of such an error surface for a radar located at  $40^\circ 52' 42''$  north latitude and  $72^\circ 41' 16''$  west longitude. This is the Riverhead radar of the NAS system, located in New York state. The system plane used for this calculation is the one used by the New York Air Route Traffic Control Center (ARTCC) with point of tangency at  $40^\circ 48' 26''$  north latitude,  $74^\circ 09' 19''$  west longitude. The error surface is created by choosing known geodetic locations and using the exact stereographic projection to determine the correct projected location. The exact range and azimuth from a specified radar site are calculated. The spherical earth approximation then provides an approximate latitude and longitude that is also projected into the system plane using the exact stereographic projection equations. The difference between the two x coordinates is plotted as the error surface. As seen from the figure, at the limits of normal radar range, the errors caused by the approximation reach a maximum of about .3 nmi, much larger than the Mode S range resolution of about .005 nmi.

In addition, if a target is placed near the range limit of each of two sensors that are 100 nmi. apart, and the system coordinates of this target are calculated using the range and azimuth as measured from each sensor, the two calculations produce different system coordinates. The magnitude of the distance between the two calculated locations varies depending on the exact geometry, but is usually greater than .1 nmi. and can reach .5 nmi in some cases. This degree of discrepancy between the system plane coordinates of a given target as calculated from different radar data is unacceptable.

#### 4.4 Correction of the Approximate Projection

From the preceding discussion, it is clear that simple application of stereographic projection to latitude and longitude values calculated from measured range, azimuth and altitude using a spherical earth approximation does not meet the needs of the Multisensor Data Processing system being developed. It is also clear that the exact solution for latitude and longitude using an ellipsoidal earth model is ruled out on the basis of the excessive amount of computation it involves. Therefore, it is desirable to make some kind of correction to the computationally tractable approximate solution. The error surface shown in Fig. 4.2 supplies a possible correction method.

Because the error surface depends only on the location of the radar and the system plane point of tangency, it is a constant over time. In addition, the error in the approximate values of latitude and longitude calculated from Eqs (4.14) changes very slowly with altitude. Therefore, a single error surface for each radar can be calculated and stored prior to system startup.

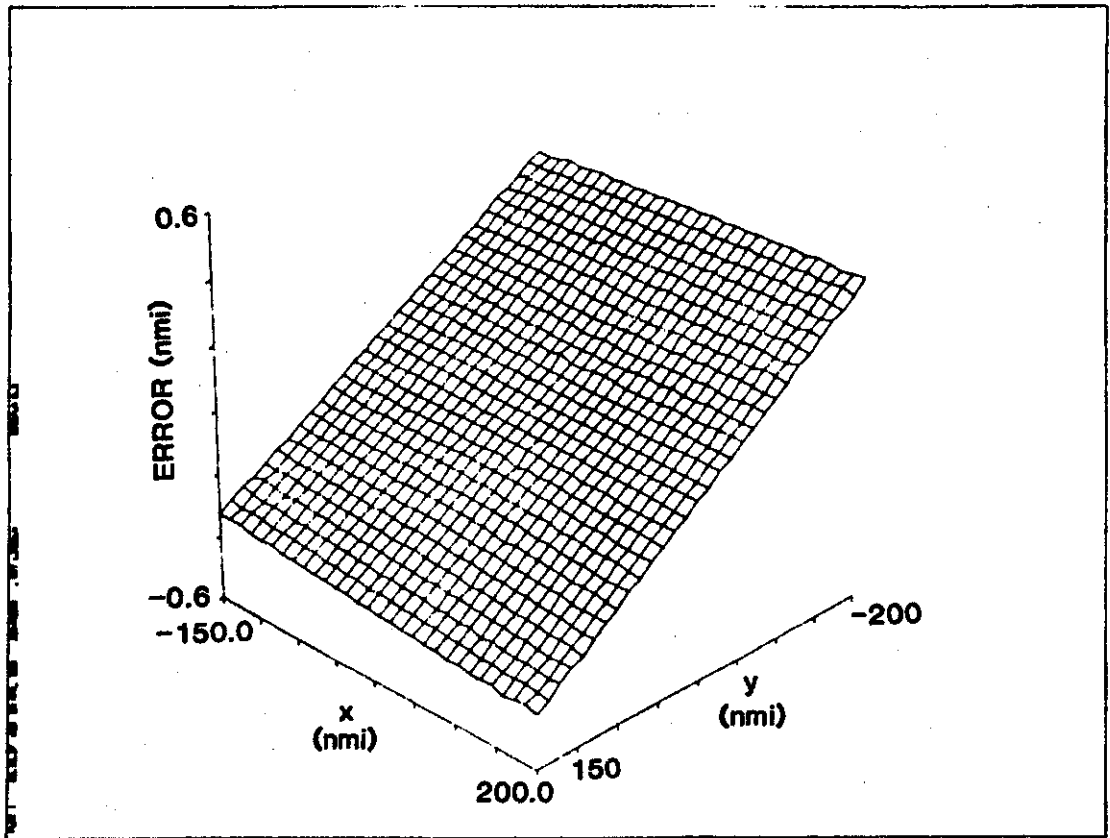


Fig. 4-2. Error in stereographic projection using a spherical earth approximation.

Then, depending on the particular needs of the system in terms of speed and accuracy, a table lookup or a two-dimensional polynomial approximation to the table can be used. For each target aircraft, the projected position is determined using the spherical earth approximation and the appropriate correction for that position is added. As will be seen in the next section, this procedure gives very good results.

## 5.0 AN IMPLEMENTATION OF CORRECTED STEREOGRAPHIC PROJECTION

The stereographic projection algorithm described in this section is one that is suggested as meeting the requirements of the Multisensor Data Processing system for both accuracy and speed. The algorithm uses the process of conversion to a local, radar-centered stereographic plane followed by transformation to the system coordinates as described in Section 4.1.1 and 4.1.2. This process uses a spherical earth approximation to increase computational efficiency. The errors introduced by this approximation are corrected using a precomputed error polynomial, as outlined in Section 4.4, to produce the final system coordinates. Following a discussion of the details of the projection process, the performance of this implementation is examined, and is found to satisfy all anticipated requirements of the Multisensor Data Processing system. The inverse projection, from system x,y coordinates to radar-centered range and azimuth is also examined briefly at the end of this section. Implementation of the inverse projection is shown to require little additional effort.

### 5.1 Projection to System Coordinates

In Section 4, two methods of conversion to stereographic coordinates were discussed. One involves projection to a local plane and subsequent transformation into the system plane, while the other calculates latitude and longitude from measured range and azimuth and then applies the exact stereographic projection to produce system plane coordinates. The second method was used in the previous section to evaluate the performance of the conversion system and to compare the results to the required Multisensor Data Processing system performance. Both methods are considered in this section on the basis of the amount of computation required to implement the method and the performance of the method when correction is applied.

The procedure using projection to a local plane followed by transformation to system coordinates requires less computation due to decreased use of trigonometric functions. Therefore, if the performance of the two methods is equivalent, this method should be used. Performance of either method will be good if the error surface generated by that method is smooth and slowly varying over the region of interest and if the error surface is independent or nearly independent of altitude. These properties guarantee that only one error surface for each radar need be calculated, and that simple interpolation techniques or low-order polynomial approximations will give valid correction values between the explicitly calculated values.

If the initial projection into a local plane is made using Eq. (4.2), the error surface is altitude independent. Numerical investigation has shown that the error surfaces from both methods are smooth, and that the performance of the two projection methods are effectively identical over a rectangular system plane up to 1200 nmi. on a side. Therefore, projection to a local stereographic plane followed by transformation to the final system coordinates is used, with Eq. (4.2) being used for initial projection.

Once the local Cartesian coordinates are determined, the formulas of Eq. (4.10) are used to transform to system coordinates. Again, as in the NAS implementation, a transition range can be determined for each sensor so that

transformation of data with a smaller range can be made using only the first order equations. However, this is not desirable because the transition from first-to second-order approximation can cause a discontinuity in the calculated error surface. This would have detrimental effects on the correction process due to difficulties in representing the transition with the simple and efficient interpolation or fitting techniques that are to be used in the correction process.

This process provides the initial approximation to the system coordinates. Correction is then applied to this initial approximation to provide the final coordinates in the stereographic system plane.

## 5.2 Correction of Systematic Projection Errors

The correction process outlined previously is used to eliminate known systematic errors caused by the use of approximations to the exact projection of an ellipsoid onto a plane. Because these errors depend on the geometry of radar location with respect to the stereographic plane point of tangency, a different correction must be made for each radar. The errors also depend on the location of the target with respect to the radar, but are nearly independent of target altitude. These dependencies make it necessary to determine, for each radar site, a two-dimensional error grid or surface that will determine the size of the correction that will be necessary for a target at any specified ground range and azimuth angle.

This error surface is calculated through a number of steps:

- 1) The coverage area for which correction is desired is defined and is divided into a grid in terms of latitude and longitude.
- 2) The chosen grid points are projected onto the stereographic plane using the exact stereographic projection of a spheroid.
- 3) Range and azimuth values are calculated for a target at a fixed altitude above the chosen grid points. These values are exact to the accuracy of the chosen earth model and of the mathematical functions (ie. trigonometric and other standard functions) available.
- 4) The algorithm described in the previous section to determine the initial approximate system coordinates is applied to the range and azimuth calculated for each grid point.
- 5) The differences between the exact and approximate system coordinates are calculated, and a two dimensional polynomial fit to nth order is made.

This process results in two polynomial approximations, one for offsets in the x direction and one for y offsets. The polynomials are of the form

$$\begin{aligned} \Delta x &= a_{00} + a_{01} y + a_{10} x + a_{11} xy + \dots + a_{mm} x^m y^m \\ \Delta y &= b_{00} + b_{01} y + b_{10} x + b_{11} xy + \dots + b_{mm} x^m y^m \end{aligned} \tag{5.1}$$

The fits are made using the Cartesian coordinates from the approximate conversion as independent variables. This allows the running real-time system to generate approximate system coordinates and to use these approximate coordinates directly in the pre-calculated polynomials to determine the proper corrections.

Once the polynomial coefficients have been determined by the polynomial fitting procedure, some of the coefficients can often be discarded. To determine which coefficients are required, it is first necessary to determine the desired accuracy of the corrected system coordinates. If the magnitude of the correction generated by a given coefficient at the maximum range of interest for a given radar is an order of magnitude less than the maximum allowed error, that coefficient can be set to zero. In the present case, since projection errors of less than about .005 nmi. are desired, the coefficients are evaluated on the basis of a .001 nmi. maximum error.

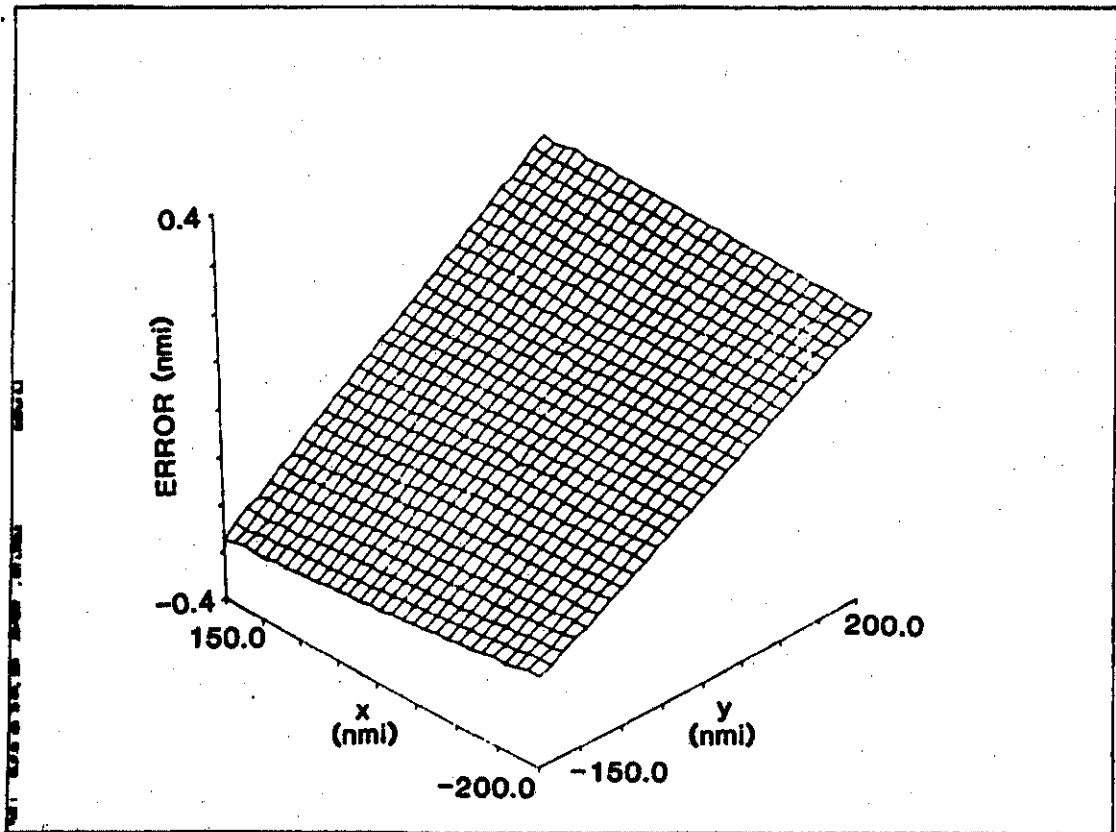
Although the polynomial evaluation used in this correction scheme requires a significant amount of computation, this correction method is far less expensive than other methods that result in comparable accuracy.

### 5.3 Performance of the Corrected Projection Algorithm

As discussed earlier, the stereographic projection, in its exact form, meets the requirements of air traffic control systems better than other available projection algorithms. In Section 4.3 it was shown that a projection algorithm based on a spherical earth model could not meet the needs of the new MDP system being developed without some form of correction. This conclusion was based on the requirement of maximum projection errors of a few thousandths of a nautical mile, and the requirement that maximum discrepancies between the system coordinates of a single target as determined by any two radars be of the same size.

The correction method recommended above meets these requirements without introducing an unacceptable computational load. The performance of the corrected algorithm can best be examined by use of error surfaces similar to those used previously. Figures 5.1 through 5.3 show the error surfaces for the same system as that used in Fig. 4.2. Figure 5.1 used the uncorrected version of the projection algorithm described earlier in this section. The errors at long range in this figure are of the order of tenths rather than thousandths of nautical miles. In Fig. 5.2, the differences between the corrected approximate projection and the exact projection are plotted. Figure 5.3 shows the corrected surface on an expanded scale. Note that now the maximum error values are less than a few thousandths of a nautical mile as required.





**Fig. 5-1. Error in the uncorrected projection algorithm.**

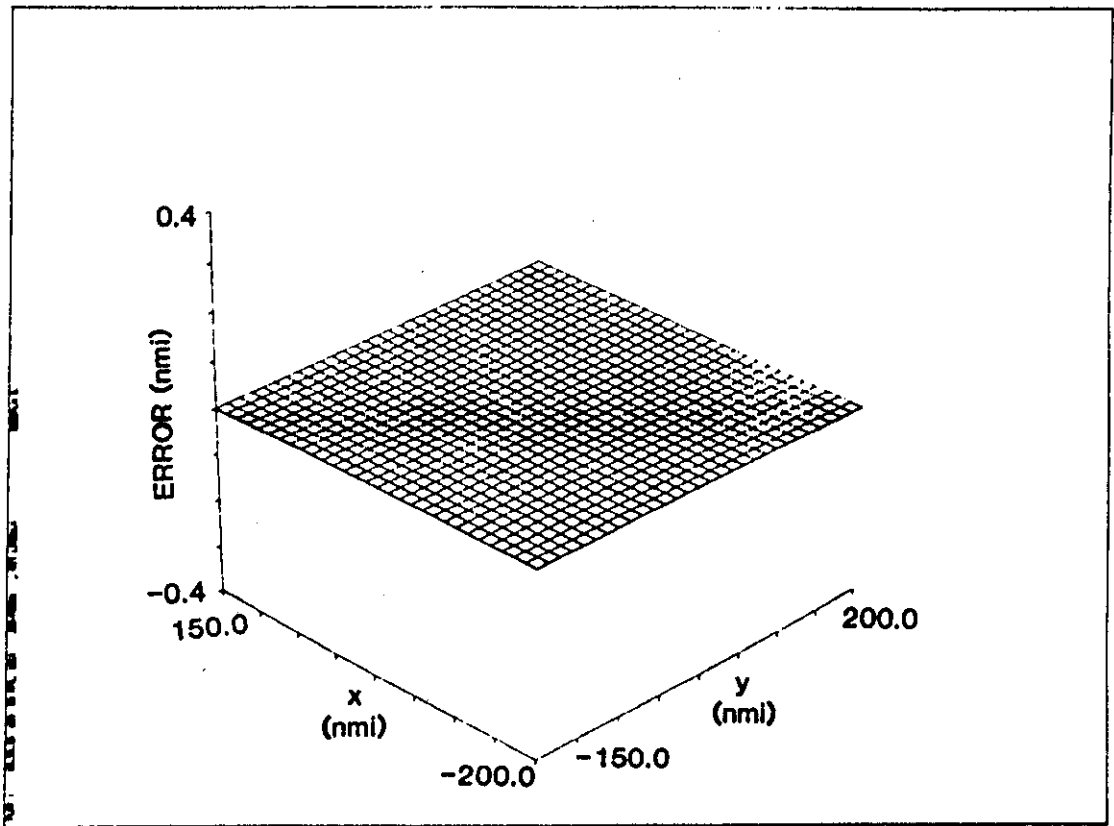


Fig. 5-2. Error in projection algorithm after correction.

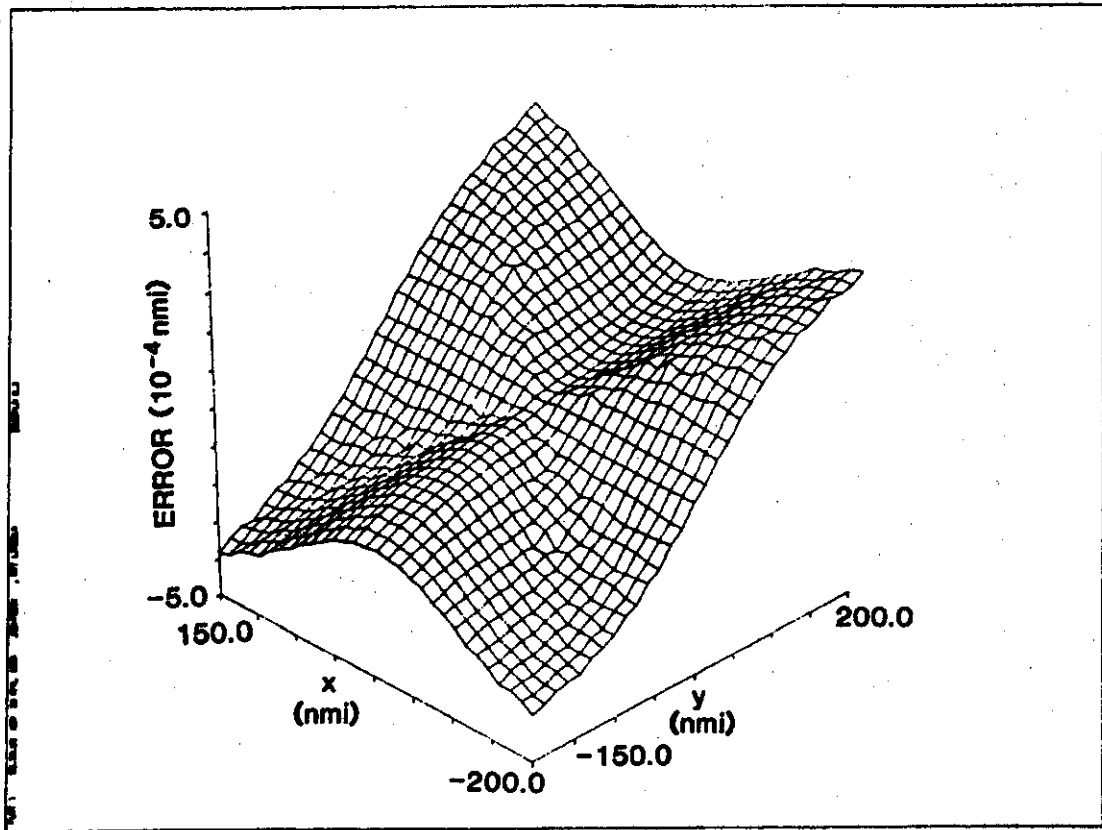


Fig. 5-3. Error after correction plotted on an expanded scale.

Figures 5.4 through 5.6 show a somewhat different error surface. This surface is the multisensor registration error, the distance between the system coordinates of a single target as determined by two radars located 100 nmi. apart. The coverage area is chosen so that the maximum range from either radar is about 150 nautical miles. It is seen in Fig. 5.4 that the maximum errors generated by the uncorrected algorithm are a few tenths of a nautical mile, again far larger than the desired error limits. When the correction is applied, and the error surface is again determined, the maximum errors are reduced to acceptable limits as seen in Figure 5.5, a plot of the corrected data on an expanded scale. Figure 5.6 shows both surfaces on the same scale for comparison. The same behavior is seen using other locations for system point of tangency and other radar site geometries.

Therefore, it is reasonable to conclude that the implementation of stereographic projection suggested here, including the correction for errors caused by use of approximations, is sufficient to meet the needs of the Multisensor Data Processing system being developed.

#### 5.4 The Inverse Projection

In the Multisensor Data Processing system, it will be necessary to take stereographic system coordinates for an aircraft track and convert back to radar-centered range and azimuth. Fortunately, the stereographic projection method in general, and the implementation described here in particular, is invertible. For this implementation, there are three separate steps that must be inverted. A transformation must be made from the system stereographic plane to a local radar plane, the local radar coordinates must be converted to range and azimuth, and a correction for approximations in the projection process must be applied.

Transformation from the system plane to the local radar plane can be made using Eq. (4.7), simply by replacing all references to system plane parameters with the corresponding local radar plane parameters and vice versa. This results in the formula for local radar  $u$  and  $v$

$$z = z_0 T' W e^{-i\beta'} + T' K' W^2 e^{-2i\beta'} e^{-i\gamma'} \quad (5.2)$$

where

$$T' = 1 + \frac{z_0 \bar{z}_0}{4E^2}$$

$$K' = \frac{\sqrt{u_0^2 + v_0^2}}{4E^2}$$

$$z_0 \bar{z}_0 = (u_0^2 + v_0^2)$$

$$\gamma' = \arctan (u_0/v_0)$$

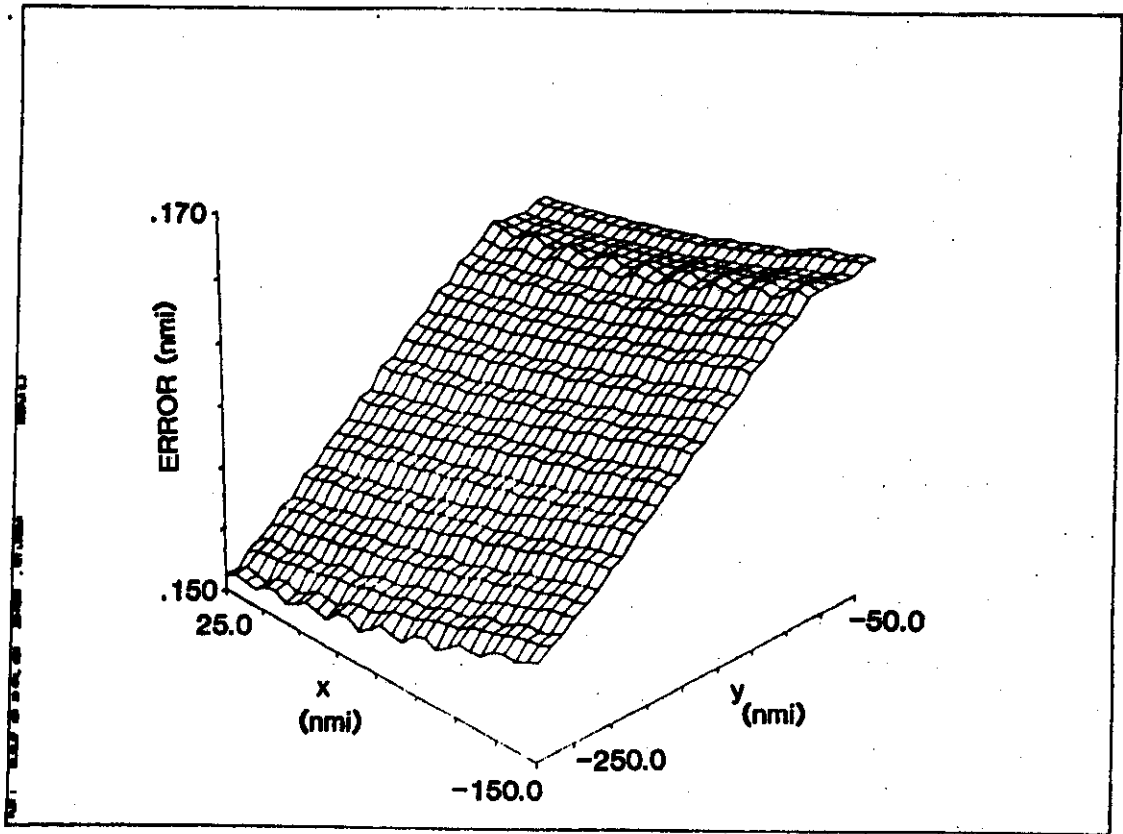
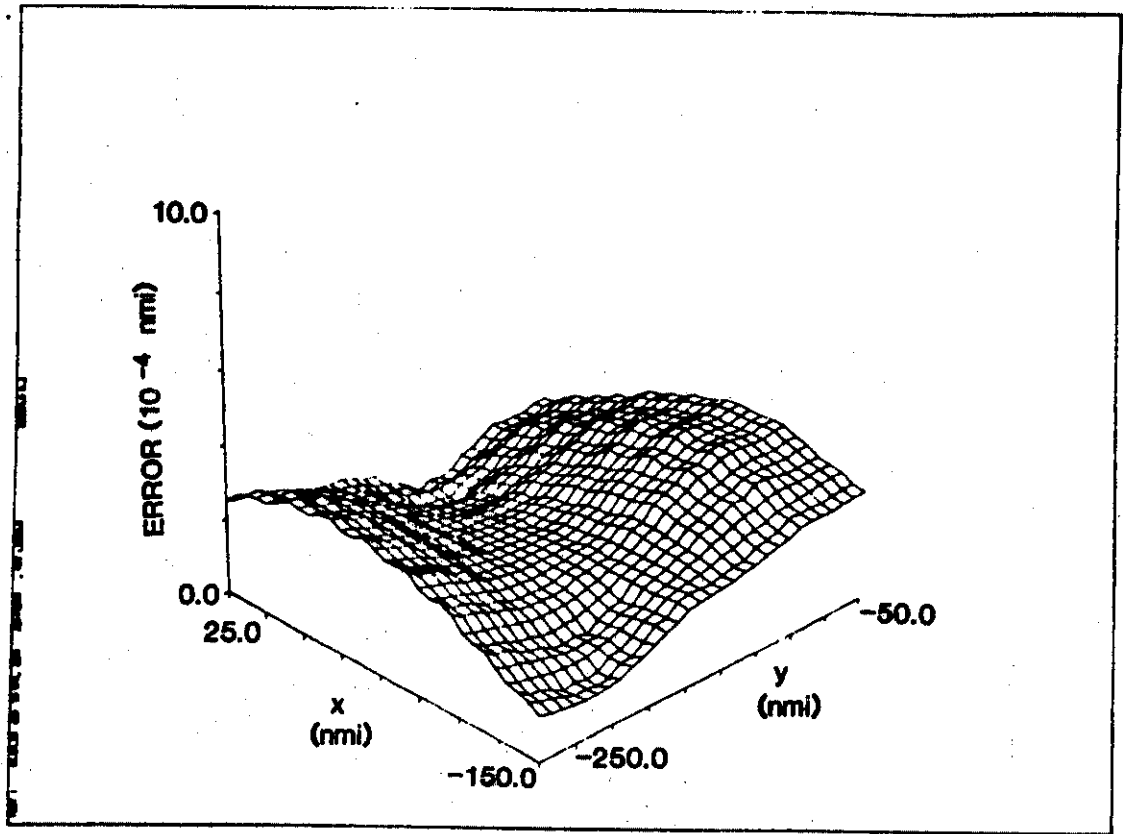


Fig. 5-4. Multisensor registration error before correction.



**Fig. 5-5. Multisensor registration error after correction.  
Note the expanded error scale.**

$$\beta' = \frac{(\sin L + \sin L_0) \sin(\lambda - \lambda_0)}{\cos(\lambda - \lambda_0) + \sin L \sin L_0 \cos(\lambda - \lambda_0) + \cos L \cos L_0}$$

Ground range and azimuth are determined from Z by

$$\begin{aligned} D &= \sqrt{u^2 + v^2} \\ \theta &= \arctan(u/v) \end{aligned} \tag{5.3}$$

Slant range is then found from

$$\begin{aligned} \phi &= 2 \arctan \left( \frac{D}{2E} \right) \\ \rho &= \left[ 2 \cos \phi (E+h)(E+H) - ((E+h)^2 + (E+H)^2) \right]^{1/2} \end{aligned} \tag{5.4}$$

Correction can be applied in a way similar to that used for the projection to system coordinates. An error surface in the local radar plane can be determined, a polynomial fit determined, and correction applied by evaluation of the error polynomial at the newly determined local x and y coordinates. Since this guarantees that the local ground range and azimuth will be correct to arbitrary accuracy, Eq. (5.4) will give the desired value for slant range and azimuth.

Therefore the inverse projection is comparable in terms of computation and accuracy to the direct stereographic projection. Conversion of stereographic system plane coordinates to radar-centered range and azimuth can be included in the system without significant additional effort.

## 6.0 SUMMARY AND CONCLUSIONS

The factors involved in the choice of a method for conversion of radar-centered range, azimuth, and altitude coordinates to a system plane using x, y, and altitude coordinates have been examined. The properties of a variety of available cartographic projection methods were then evaluated. As a result of this process, a choice of the stereographic projection as the preferred projection method has been made. This choice is consistent with that previously made for use in the current NAS multiple radar data processing system.

Having decided on a projection technique, more specific implementation details were examined. In particular, differences in requirements between the running NAS system and the Multisensor Data Processing system under development were examined. In light of the increased requirements of the new system, it was determined that the NAS implementation is not sufficient. It was also determined that any projection implementation based on a spherical or locally spherical model of the earth would be insufficient to meet the requirements of the new system. Therefore, any practical implementation would require either the use of a more complex ellipsoidal earth model, or application of some form of correction to the spherical earth approximation.

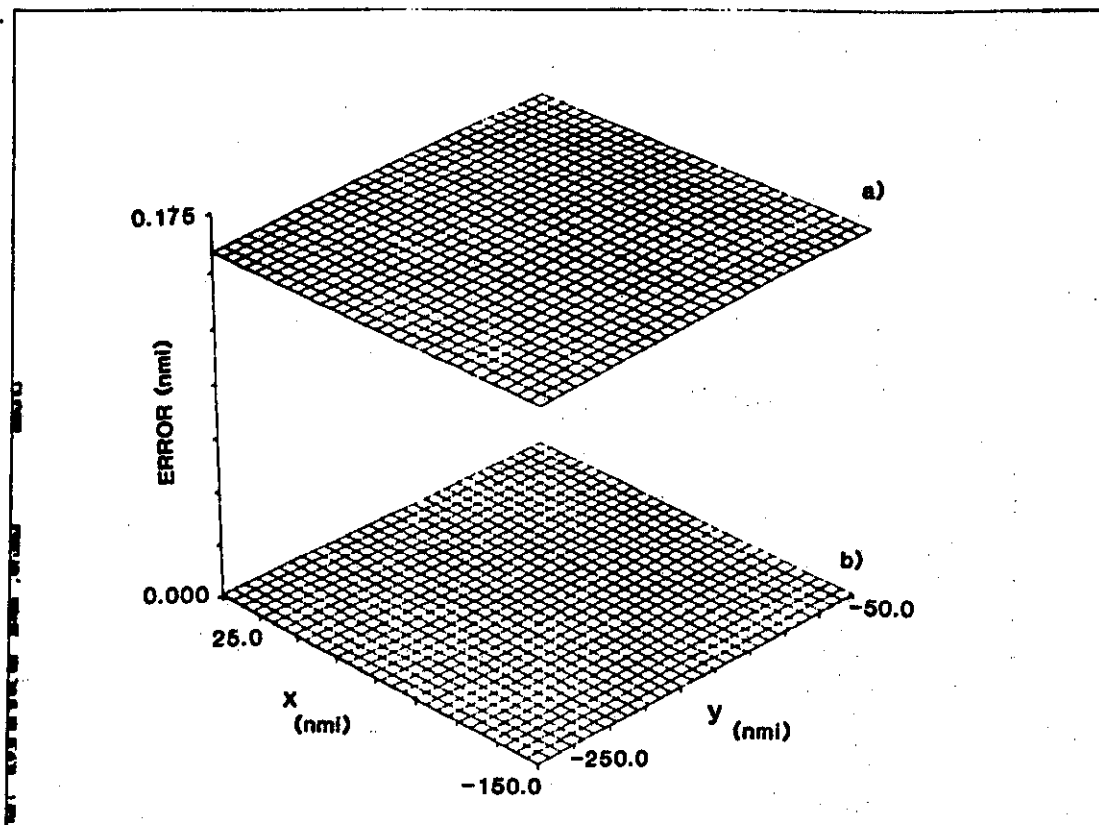
The computational requirements for use of an accurate ellipsoidal earth model proved to be prohibitive due to the necessity of using iterative solution techniques. Therefore a method for correcting the spherical earth model was developed. This method uses a pre-computed polynomial approximation to a two-dimensional error surface to correct initial stereographic coordinates calculated using a locally spherical earth model. This correction scheme has been tested and found to meet the accuracy requirements of the new Multisensor Data Processing system for which it is designed.

In conclusion, a coordinate conversion algorithm has been designed and tested. The conversion process does introduce a certain amount of magnification error, but this is unavoidable when projecting an ellipsoidal surface onto a planar one. The magnification errors are acceptable because they are small, remaining less than 1% for practical control center coverage areas, and because any error in distances between two aircraft approaches zero as the separation approaches zero. The chosen algorithm preserves angular validity and thus maintains the validity of aircraft track heading throughout the projection process. The algorithm that has been designed and described here meets all of the currently anticipated needs of the Multisensor Data Processing system.



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**Fig. 5-6. Comparison of multisensor registration error.**  
**a) Before correction   b) After correction**